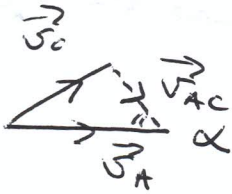


Outward

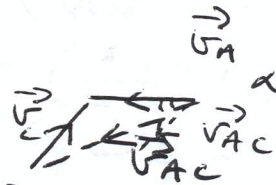


$$\vec{u}_A = u_1 \vec{i}$$

$$\vec{u}_C = \frac{v}{\sqrt{2}} \vec{i} + \frac{v}{\sqrt{2}} \vec{j}$$

$$\vec{u}_{AC} = x \cos \alpha \vec{i} - x \sin \alpha \vec{j}$$

Return



$$\vec{u}_A = -u_2 \vec{i}$$

(N.B. the minus)

$$\vec{u}_C = \frac{v}{\sqrt{2}} \vec{i} + \frac{v}{\sqrt{2}} \vec{j}$$

$$\vec{u}_{AC} = -x \cos \alpha \vec{i} - x \sin \alpha \vec{j}$$

In BOTH cases

$$\vec{u}_A = \vec{u}_{AC} + \vec{u}_C$$

$$\Rightarrow u_1 \vec{i} = x \cos \alpha \vec{i} - x \sin \alpha \vec{j} + \frac{v}{\sqrt{2}} \vec{i} + \frac{v}{\sqrt{2}} \vec{j}$$

$$\Rightarrow -u_2 \vec{i} = -x \cos \alpha \vec{i} - x \sin \alpha \vec{j} + \frac{v}{\sqrt{2}} \vec{i} + \frac{v}{\sqrt{2}} \vec{j}$$

Eqns: $u_1 = x \cos \alpha + \frac{v}{\sqrt{2}}$ (1)

Eqn: $-u_2 = -x \cos \alpha + \frac{v}{\sqrt{2}}$ (*)

Eqn: $0 = -x \sin \alpha + \frac{v}{\sqrt{2}}$ (2)

Eqn: $0 = -x \sin \alpha + \frac{v}{\sqrt{2}}$ (**)

We now have to use (1) (2) (*) and (**) to find what we were asked!

Note: (2) and (**) tell us that angle headed off in is definitely the same in both parts of the journey and

that $\sin \alpha = \frac{v}{x\sqrt{2}}$ (H)

Next to show $u_1 = u_2 = v\sqrt{2}$ and $u_1 u_2 = x^2 - v^2$ we just use (1) and (*)

Add (1) and (*) $\Rightarrow u_1 + (-u_2) = x \cos \alpha + \frac{v}{\sqrt{2}} - x \cos \alpha + \frac{v}{\sqrt{2}}$

$$\Rightarrow u_1 - u_2 = \frac{2v}{\sqrt{2}}$$

$$\Rightarrow u_1 - u_2 = \sqrt{2}v \quad \text{qed.}$$

Second:

$$u_1 = x \cos \alpha + \frac{v}{\sqrt{2}}$$

$$u_2 = x \cos \alpha - \frac{v}{\sqrt{2}}$$

$$\Rightarrow u_1 u_2 = \left(x \cos \alpha + \frac{v}{\sqrt{2}}\right) \left(x \cos \alpha - \frac{v}{\sqrt{2}}\right)$$

$$= (x \cos \alpha)^2 - \left(\frac{v}{\sqrt{2}}\right)^2 \quad (\text{because } (a+b)(a-b) = a^2 - b^2)$$